# EE 435

## Lecture 26

## Data Converter  Performance Characterization

Review from last lecture

# Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

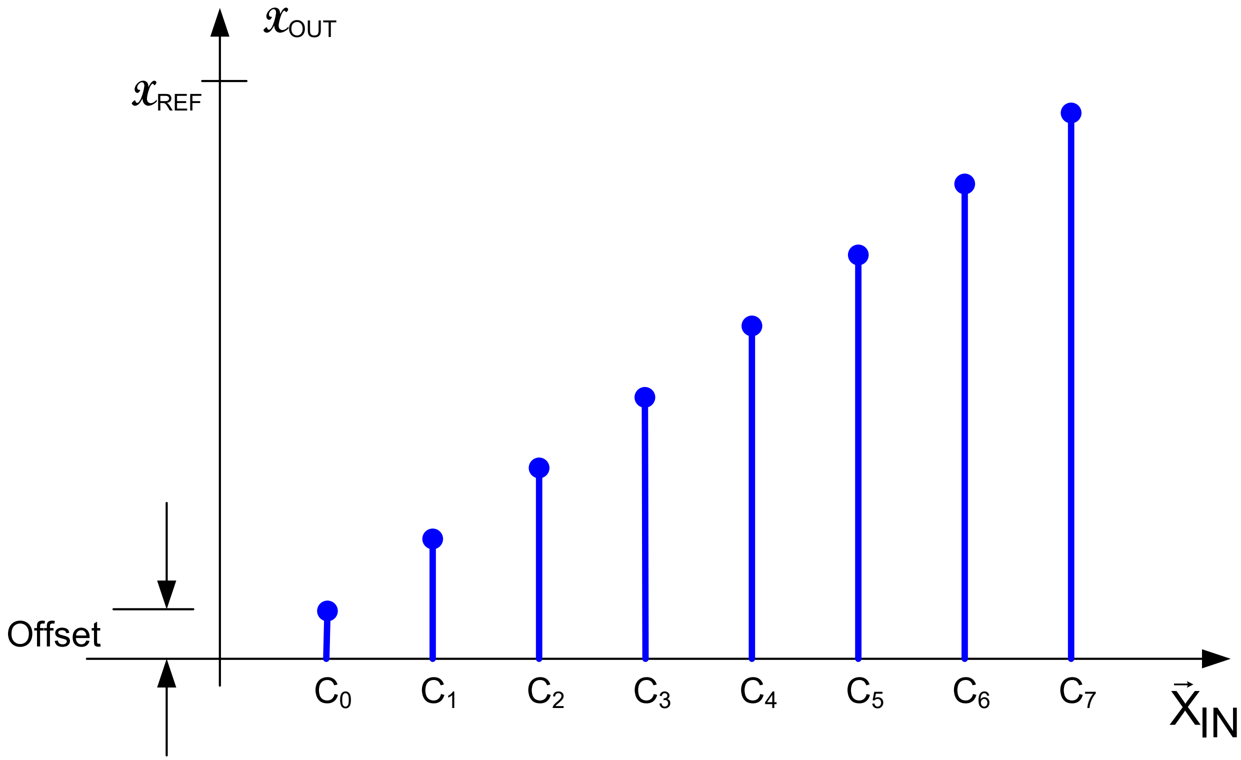
Review from last lecture

# Performance Characterization of Data Converters

- Dynamic characteristics
  - Conversion Time or Conversion Rate (ADC)
  - Settling time or Clock Rate (DAC)
  - Sampling Time Uncertainty (aperture uncertainty or aperture jitter)
  - Dynamic Range
  - Spurious Free Dynamic Range (SFDR)
  - Total Harmonic Distortion (THD)
  - Signal to Noise Ratio (SNR)
  - Signal to Noise and Distortion Ratio (SNDR)
  - Sparkle Characteristics
  - Effective Number of Bits (ENOB)

# Performance Characterization

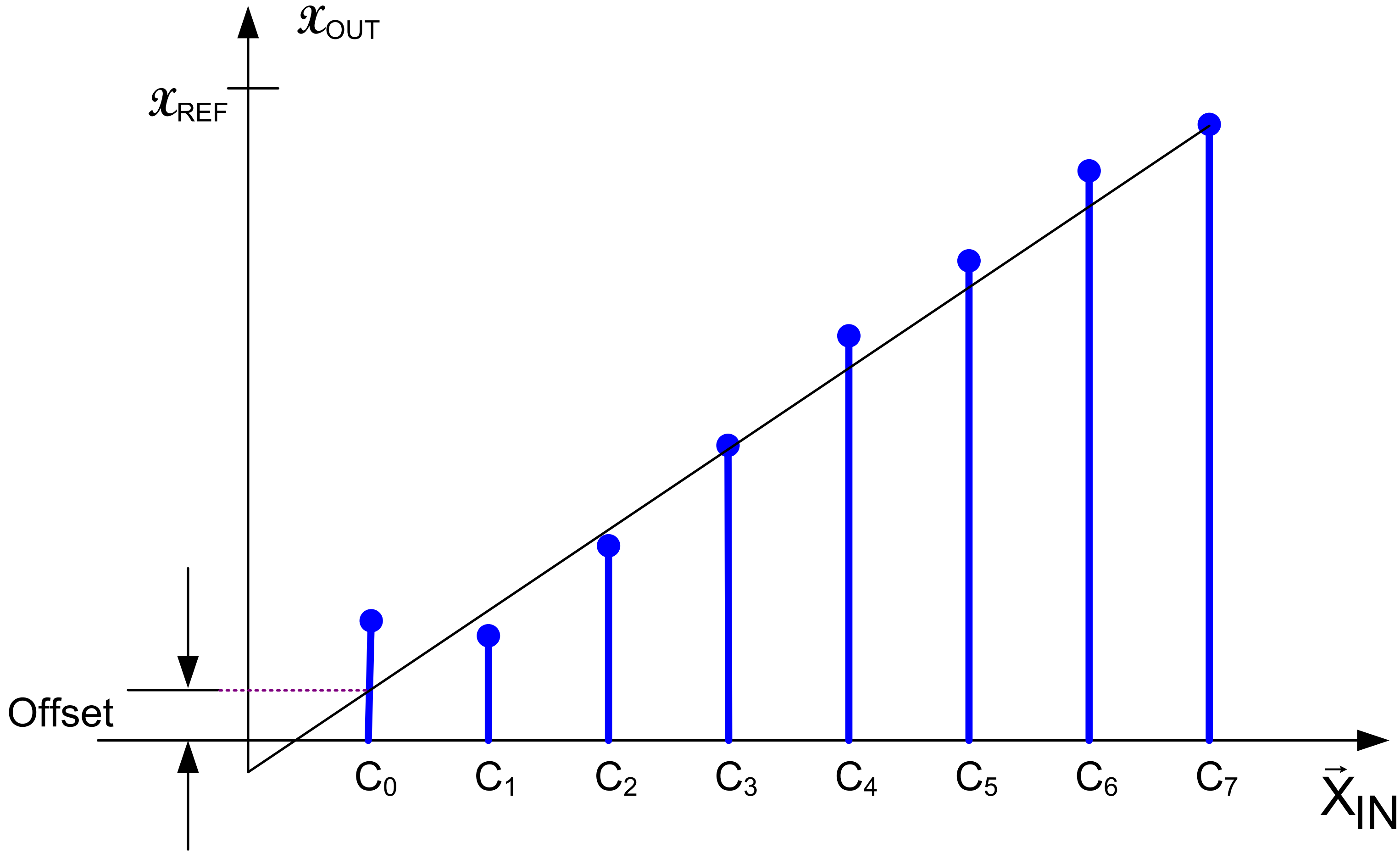
## **Offset** (for DAC)




- Offset strongly (totally) dependent upon performance at a single point
- Probably more useful to define relative to a fit of the data

# Performance Characterization

## Offset (for DAC)

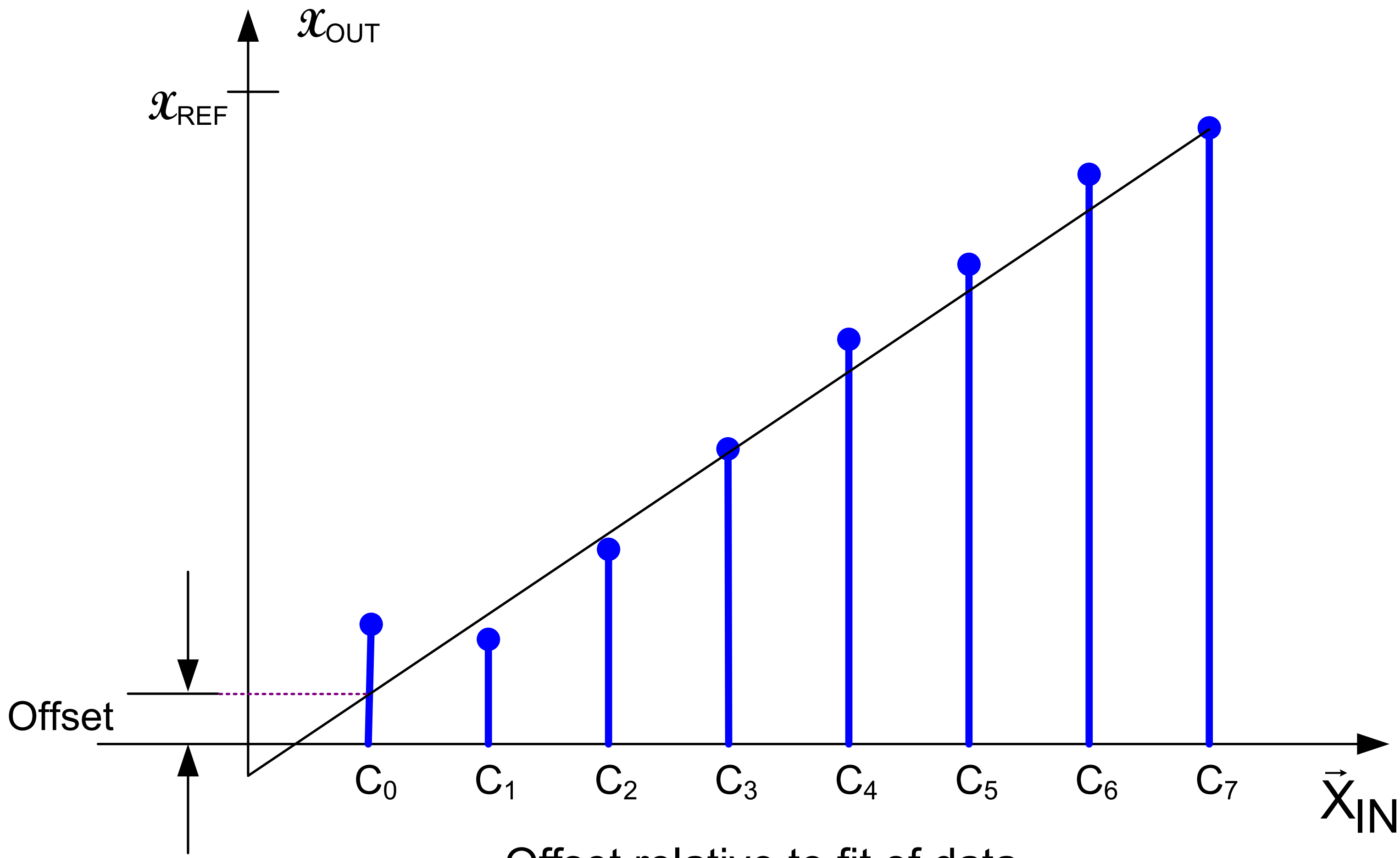



Offset relative to fit of data

Review from last lecture

# Performance Characterization

## Offset (for DAC)



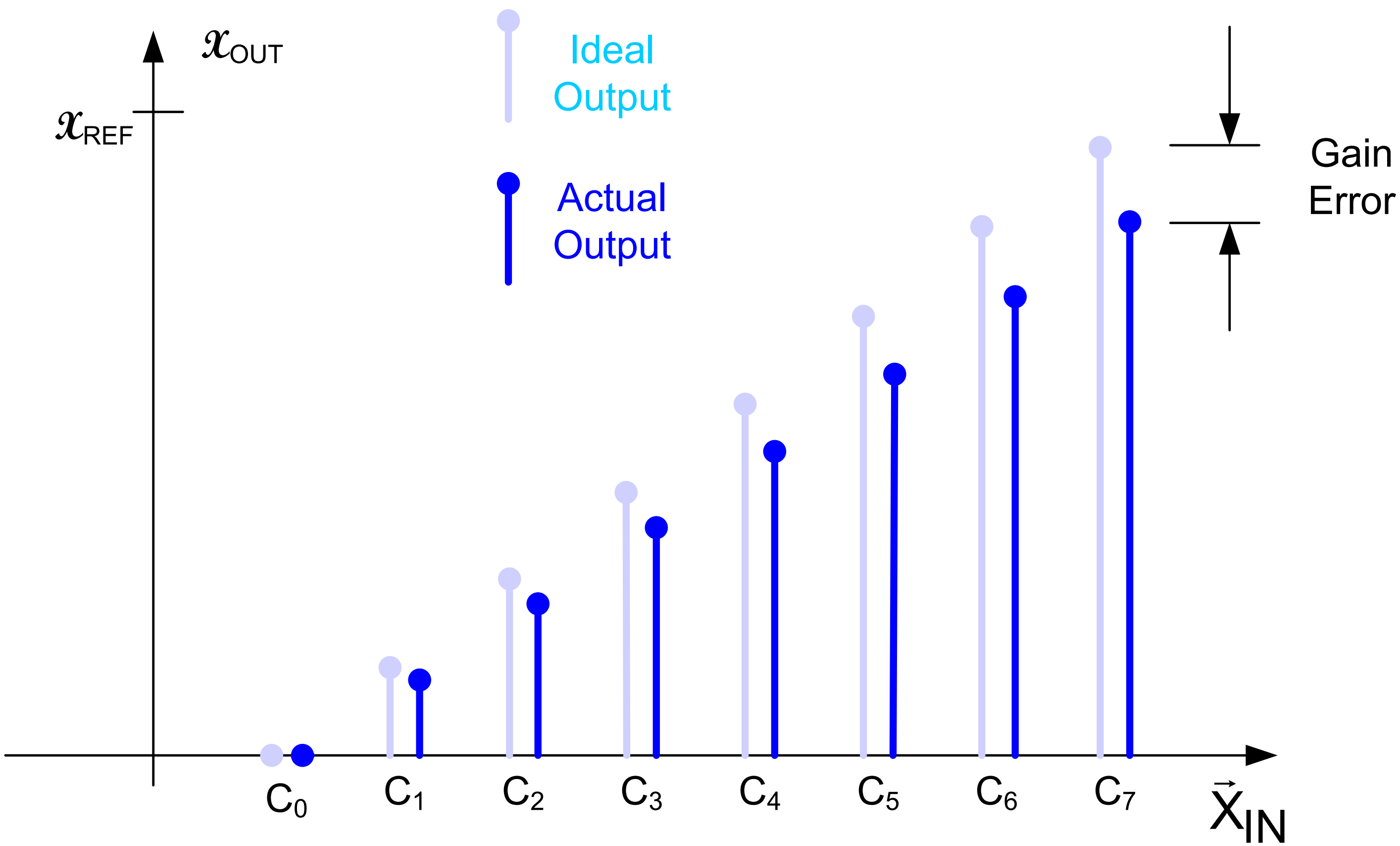

Offset relative to fit of data

Though usually more useful, not standard (more challenging to test)

# Performance Characterization

## Gain and Gain Error

For DAC




Gain error determined after offset is subtracted from output

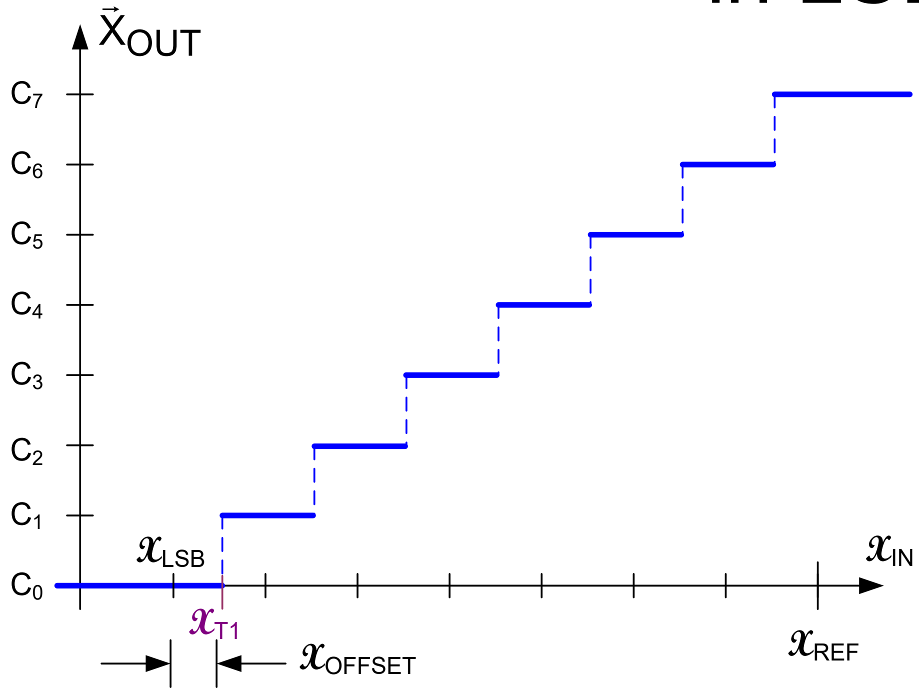
# Performance Characterization

## Offset

For ADC the offset is (assuming $\mathcal{X}_{LSB}$ is the ideal first transition point)

$\mathcal{X}_{T1} - \mathcal{X}_{LSB}$ - absolute

$$\frac{\mathcal{X}_{T1} - \mathcal{X}_{LSB}}{\mathcal{X}_{LSB}}$$ - in LSB

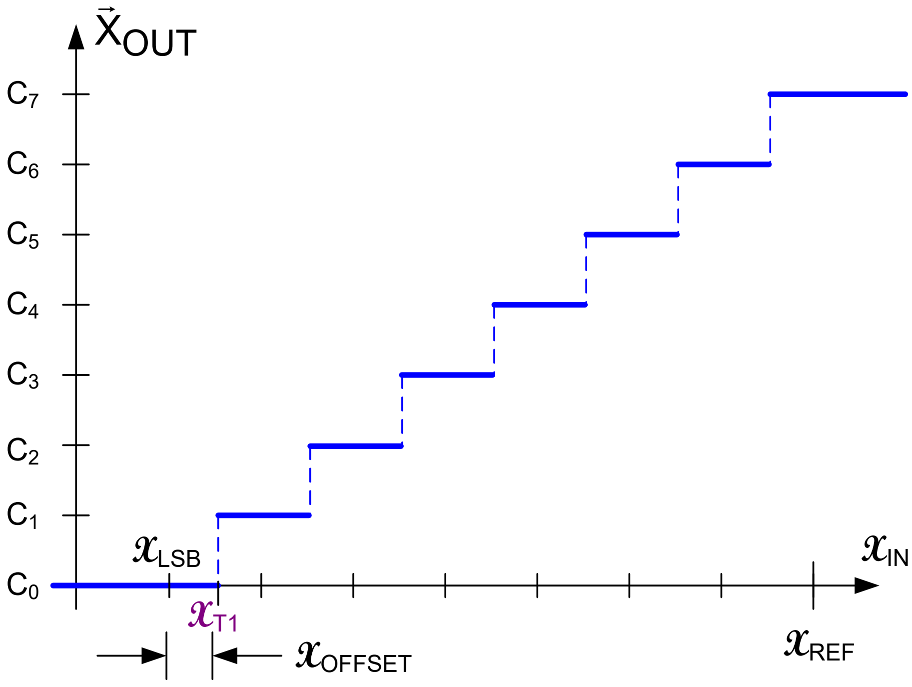



(If ideal first transition point is not $\mathcal{X}_{LSB}$, offset is shift from ideal)

# Performance Characterization

## Offset

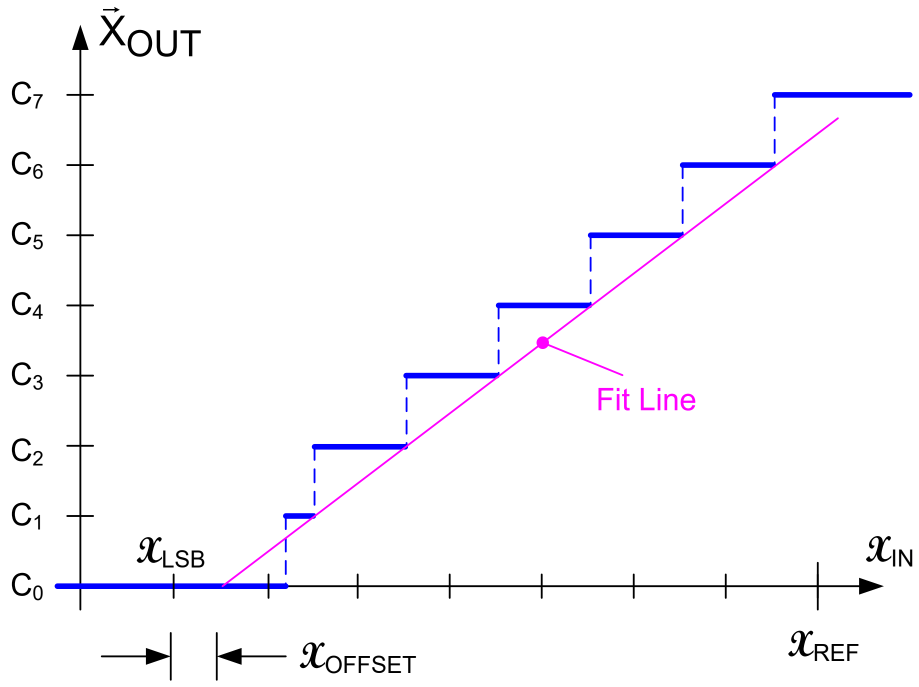
For ADC the offset is




- Offset strongly (totally) dependent upon performance at a single point
- Probably more useful to define relative to a fit of the data

# Offset

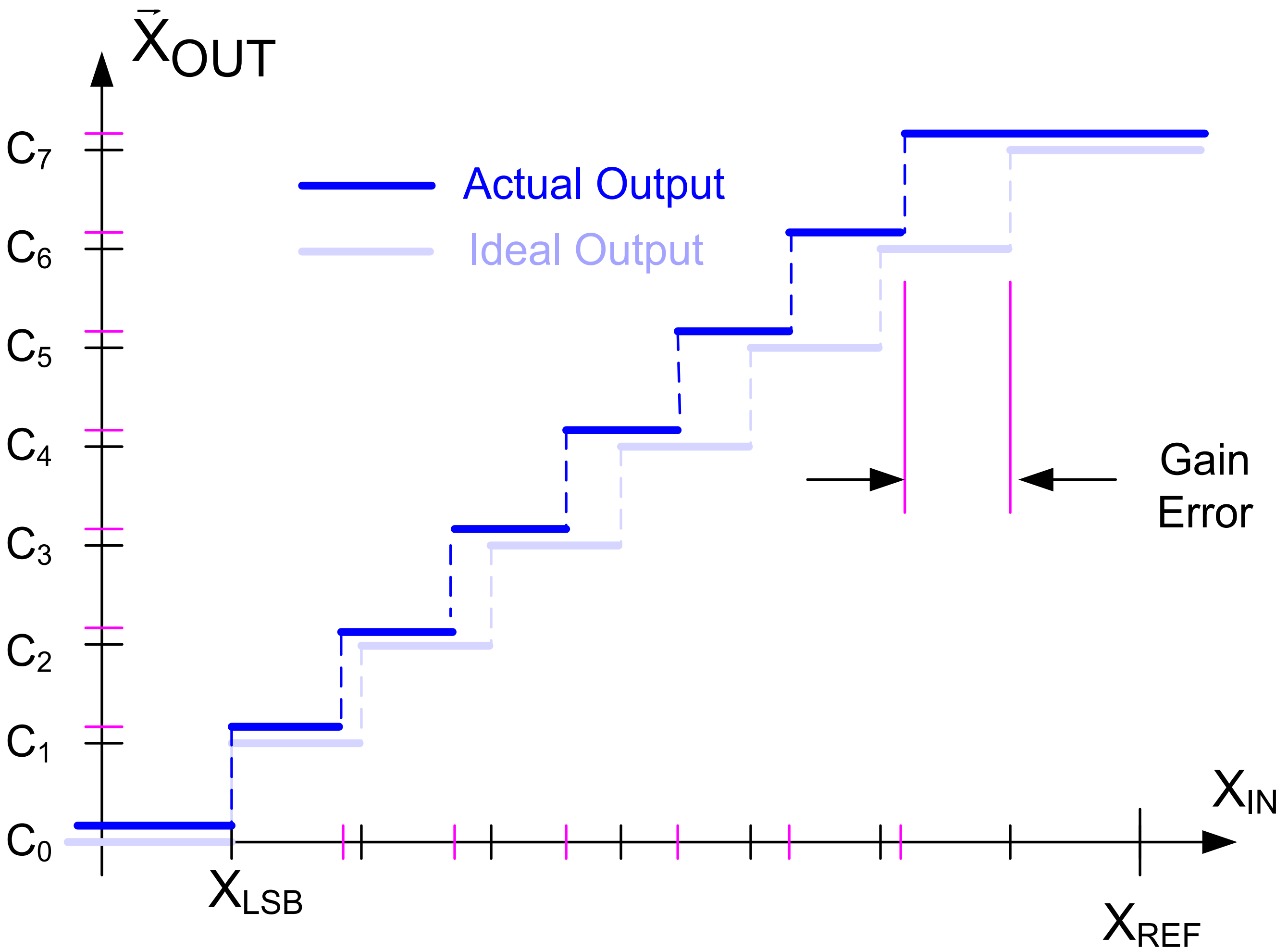
For ADC the offset is




Offset relative to fit of data

# Gain and Gain Error

For ADC



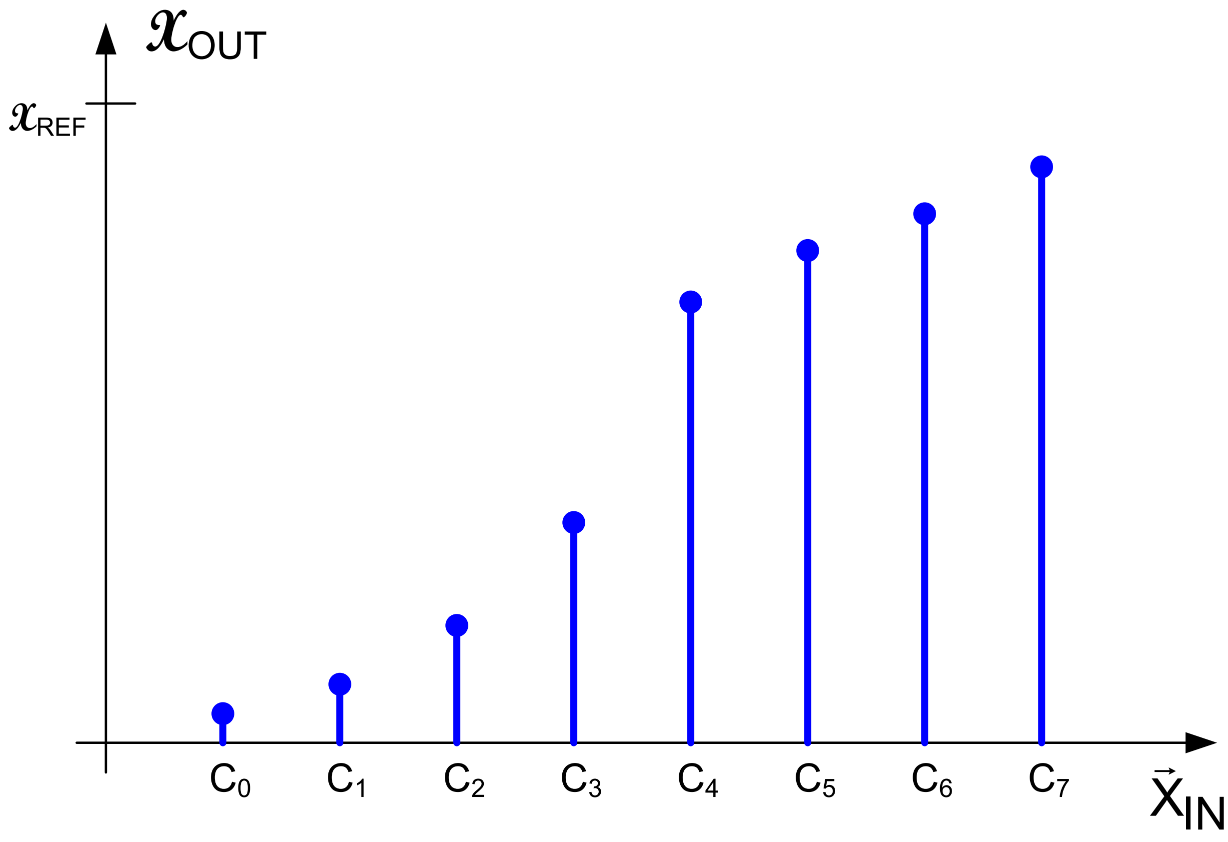

Gain error determined after offset is subtracted from output

# Gain and Offset Errors

- Fit line would give better indicator of error in gain but less practical to obtain in test
- Gain and Offset errors of little concern in many applications
- Performance characteristic of interest often nearly independent of gain and offset errors
- Can be trimmed in field if gain or offset errors exist.

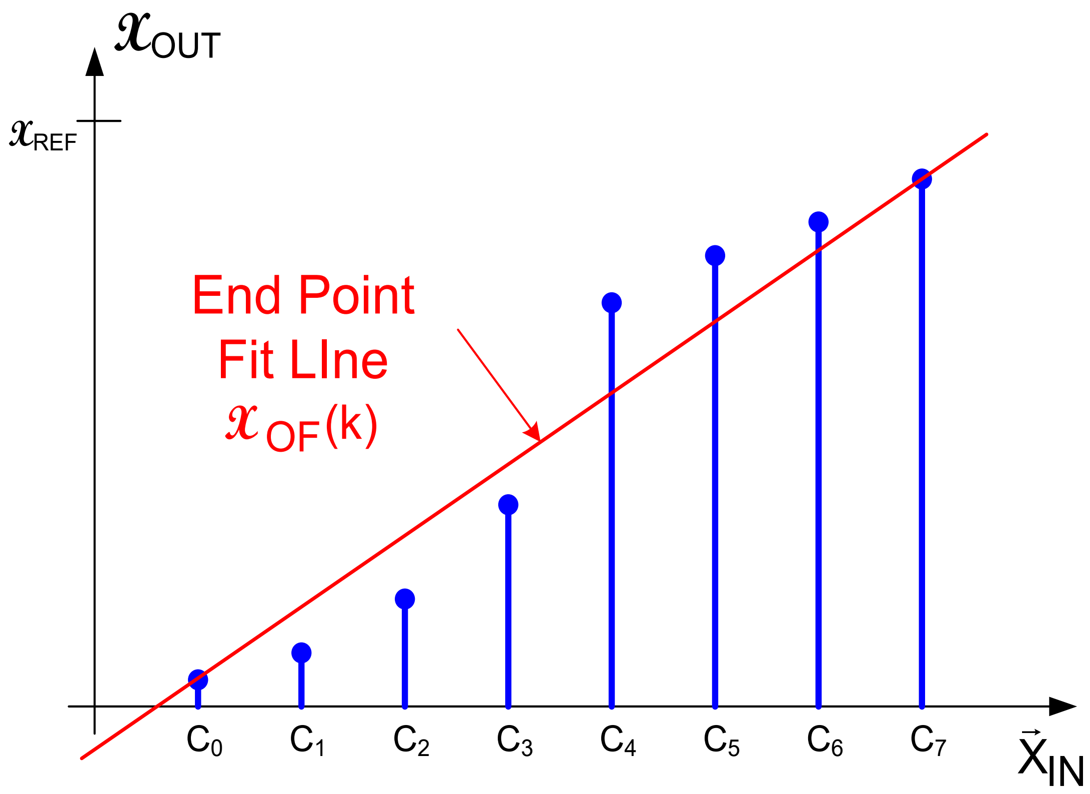
# Integral Nonlinearity (DAC)

Nonideal DAC



# Integral Nonlinearity (DAC)

Nonideal DAC

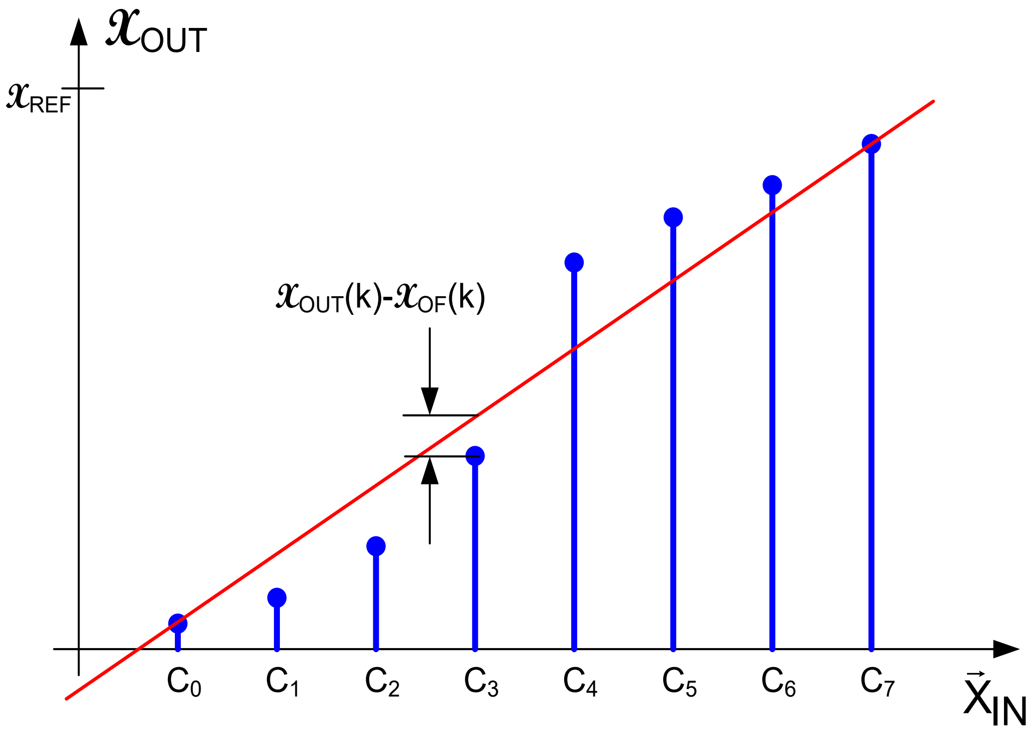



$$\mathcal{X}_{OF}(k) = mk + \mathcal{X}_{OUT}(0)$$

$$m = \frac{\mathcal{X}_{OUT}(N-1) - \mathcal{X}_{OUT}(0)}{N-1}$$

# Integral Nonlinearity (DAC)

Nonideal DAC

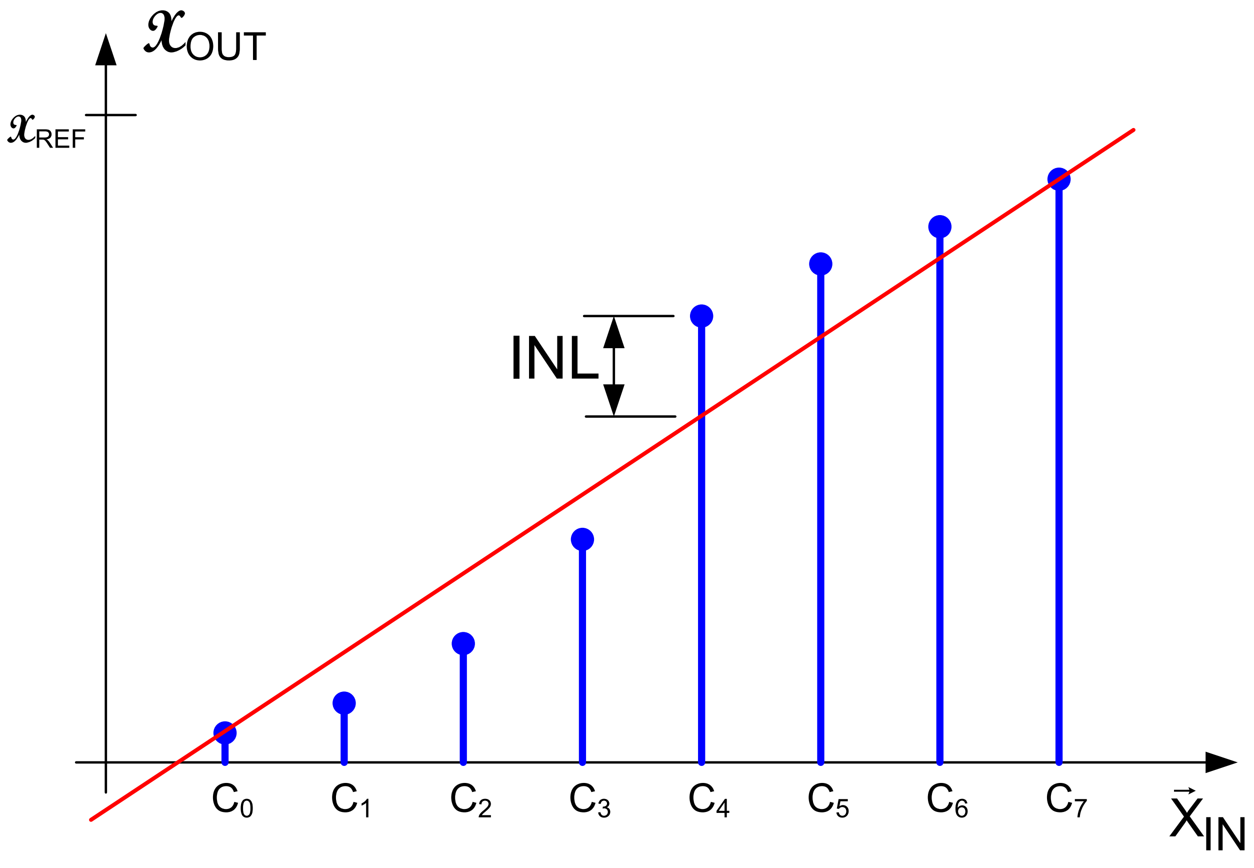



$$INL_k = x_{OUT}(k) - x_{OF}(k)$$

$$INL = \max_{0 \le k \le N-1} \{|INL_k|\}$$

# Integral Nonlinearity (DAC)

Nonideal DAC



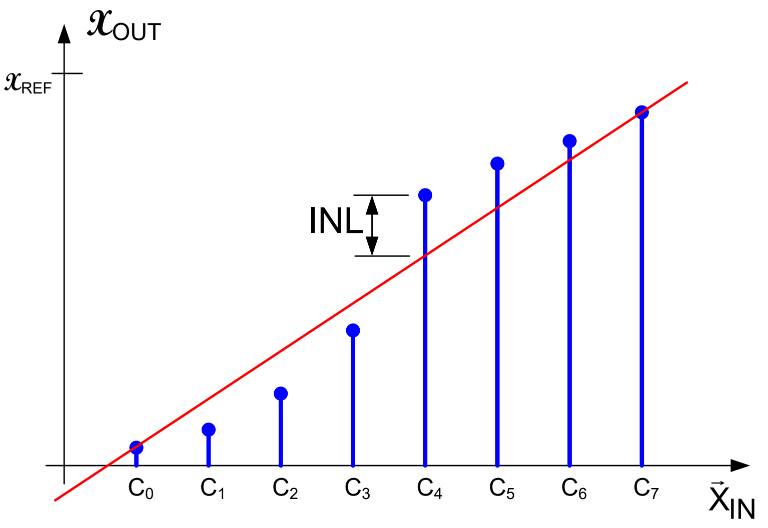
# Integral Nonlinearity (DAC)

Nonideal DAC

INL often expressed in LSB

$$INL_k = \frac{x_{OUT}(k) - x_{OF}(k)}{x_{LSB}}$$

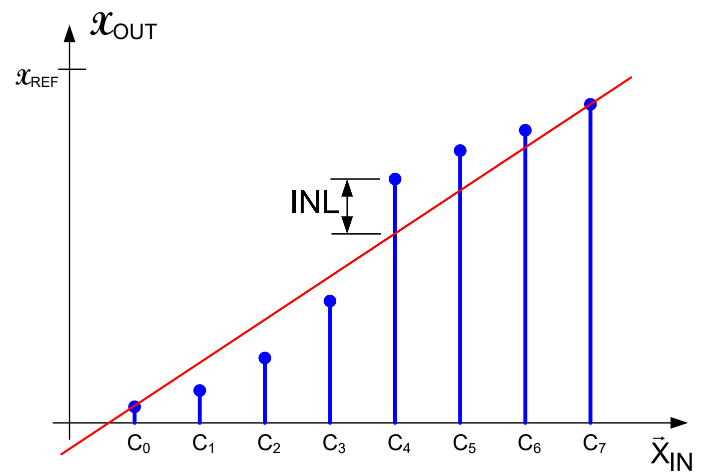
$$INL = \max_{0 \le k \le N-1} \{ |INL_k| \}$$




- INL is often the most important parameter of a DAC
- $INL_0$ and $INL_{N-1}$ are 0 (by definition)
- There are N-2 elements in the set of $INL_k$ that are of concern
- INL is almost always nominally 0 (i.e. designers try to make it 0)
- INL is a random variable at the design stage
- $INL_k$ is a random variable for $0<k<N-1$
- $INL_k$ and $INL_{k+j}$ are almost always correlated for all k,j (not incl 0, N-1)
- Fit Line is a random variable
- INL is the N-2 order statistic of a set of N-2 correlated random variables

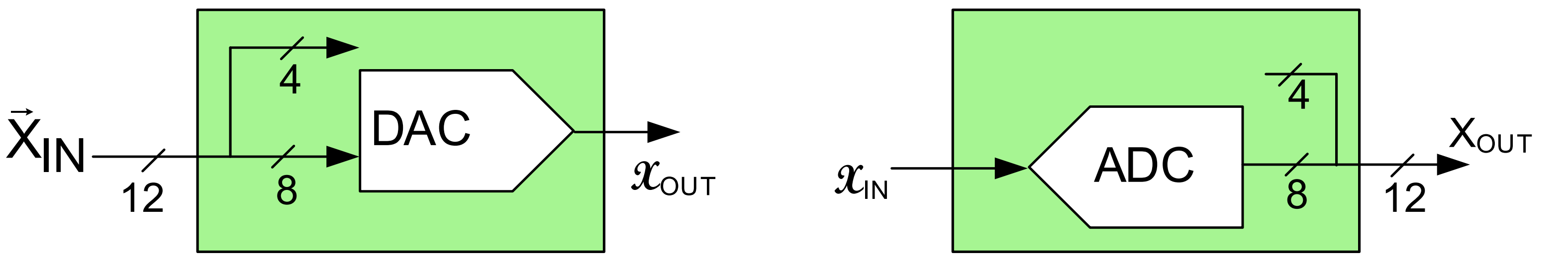
# Integral Nonlinearity (DAC)

Nonideal DAC




- At design stage, INL characterized by standard deviation of the random variable
- Closed-form expressions for INL almost never exist because PDF of order statistics of correlated random variables is extremely complicated
- Simulation of INL very time consuming if n is very large (large sample size required to establish reasonable level of confidence)
  - Model parameters become random variables
  - Process parameters affect multiple model parameters causing model parameter correlation
  - Simulation times can become very large
- INL can be readily measured in laboratory but often dominates test costs because of number of measurements needed when n is large
- Expected value of $INL_k$ at k=(N-1)/2 is largest for many architectures
- Major effort in DAC design is in obtaining acceptable INL yield !

# How many bits in this DAC?
# How many bits in this ADC?



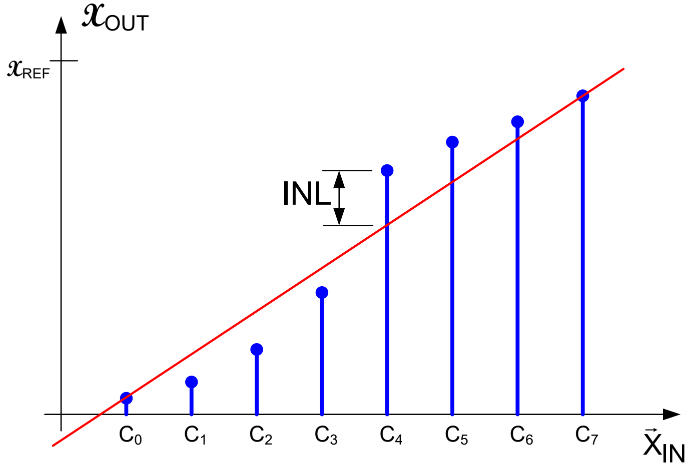

Could even have random number generator generating 4 MSBs in this ADC

Manufacturers can “play games” with characterizing data converters

That is one of the major reasons it is not sufficient to simply specify the number of bits of resolution to characterize data converters !

# ENOB of DAC

Nonideal DAC

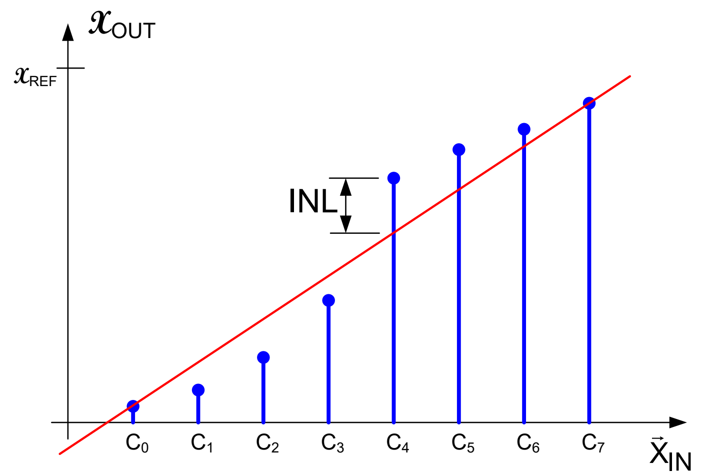



- Concept of Equivalent Number of Bits (ENOB) is to assess performance of an actual DAC to that of an ideal DAC at an "equivalent" resolution level
- Several different definitions of ENOB exist for a DAC
- Here will define ENOB as determined by the actual INL performance
- Will use subscript to define this ENOB, e.g. $\text{ENOB}_{\text{INL}}$

# $\text{ENOB}_{\text{INL}}$ of DAC

Nonideal DAC

Premise: A good DAC is often designed so that the INL is equal to ½ LSB. Thus will assume that if an n-bit DAC has an INL of ½ LSB that the $\text{ENOB}_{\text{INL}}=n$.




Hence, for "good" DAC $\quad \dfrac{INL}{V_{REF}} = \dfrac{1}{2} \bullet \dfrac{1}{2^n} = \dfrac{1}{2^{n+1}}$

Thus define the effective number of bits, $n_{EFF}$ by the expression

$$\frac{INL}{V_{REF}} = \frac{1}{2} \bullet \frac{1}{2^{n_{EFF}}} = \frac{1}{2^{n_{EFF}+1}} \quad \Longrightarrow \quad n_{EFF} = ENOB_{INL} = \log_2\left(\frac{V_{REF}}{INL}\right) - 1$$

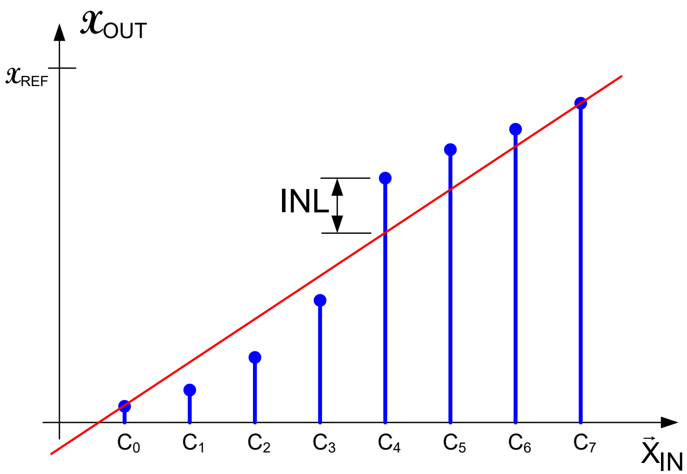
where INL is in volts

Thus, if an n-bit DAC has an INL of ½ LSB

$$ENOB_{INL} = \log_2\left(\frac{V_{REF}}{INL}\right) - 1 = \log_2\left(\frac{2^n V_{LSB}}{\frac{V_{LSB}}{2}}\right) - 1 = \log_2\left(2^{n+1}\right) - 1 = n$$

# $ENOB_{INL}$ of DAC

Nonideal DAC




Premise: A good DAC is often designed so that the INL is equal to ½ LSB. Thus will assume that if an n-bit DAC has an INL of ½ LSB that the $ENOB_{INL}$=n.

Thus, if an n-bit DAC has an INL of ½ LSB

$$ENOB_{INL} = \log_2\left(\frac{V_{REF}}{INL}\right) - 1 = \log_2\left(\frac{2^n V_{LSB}}{\frac{V_{LSB}}{2}}\right) - 1 = \log_2\left(2^{n+1}\right) - 1 = n$$

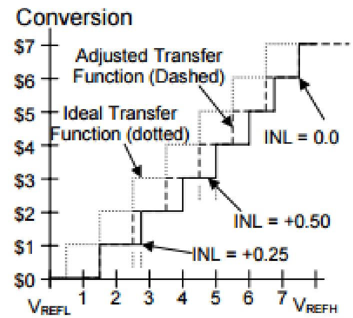
Note: With this definition, an n-bit DAC could actually have an $ENOB_{INL}$ larger than n

# Integral Nonlinearity (ADC)



**Integral Non-Linearity (INL)**

Integral Non-Linearity




Input Voltage in LSB

*Integral Non-Linearity (INL)* is defined as the sum from the first to the current conversion (integral) of the non-linearity at each code (*Code DNL*). For example, if the sum of the DNL up to a particular point is 1LSB, it means the total of the code widths to that point is 1LSB greater than the sum of the ideal code widths. Therefore, the current point will convert one code lower than the ideal conversion.

In more fundamental terms, INL represents the curvature in the Actual Transfer Function relative to a baseline transfer function, or the difference between the current and the ideal transition voltages. There are three primary definitions of INL in common use. They all have the same fundamental definition except they are measured against different transfer functions. This fundamental definition is:

Code INL = V(Current Transition) – V(Baseline Transition)

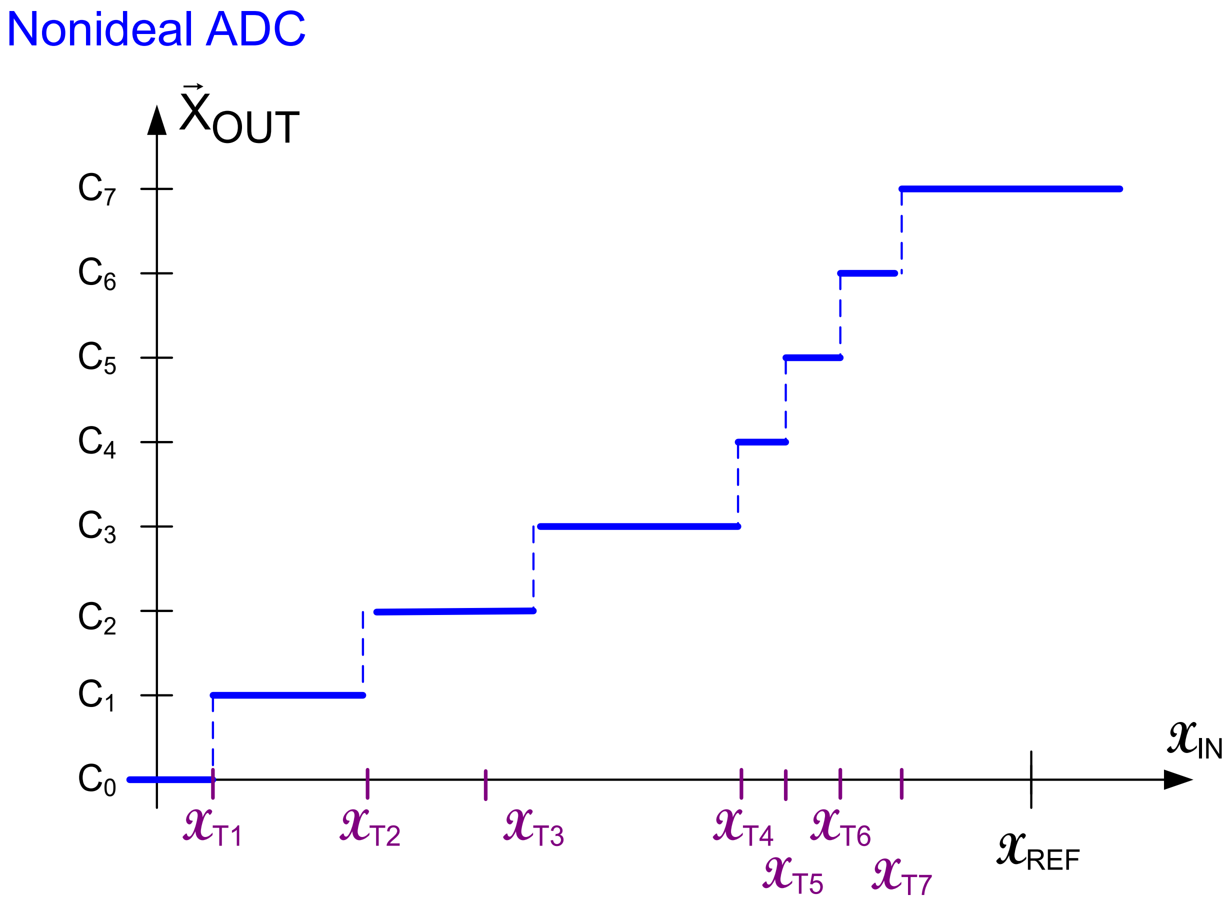
INL = Max(Code INL)





Actually probably more than 3

# Integral Nonlinearity (ADC)

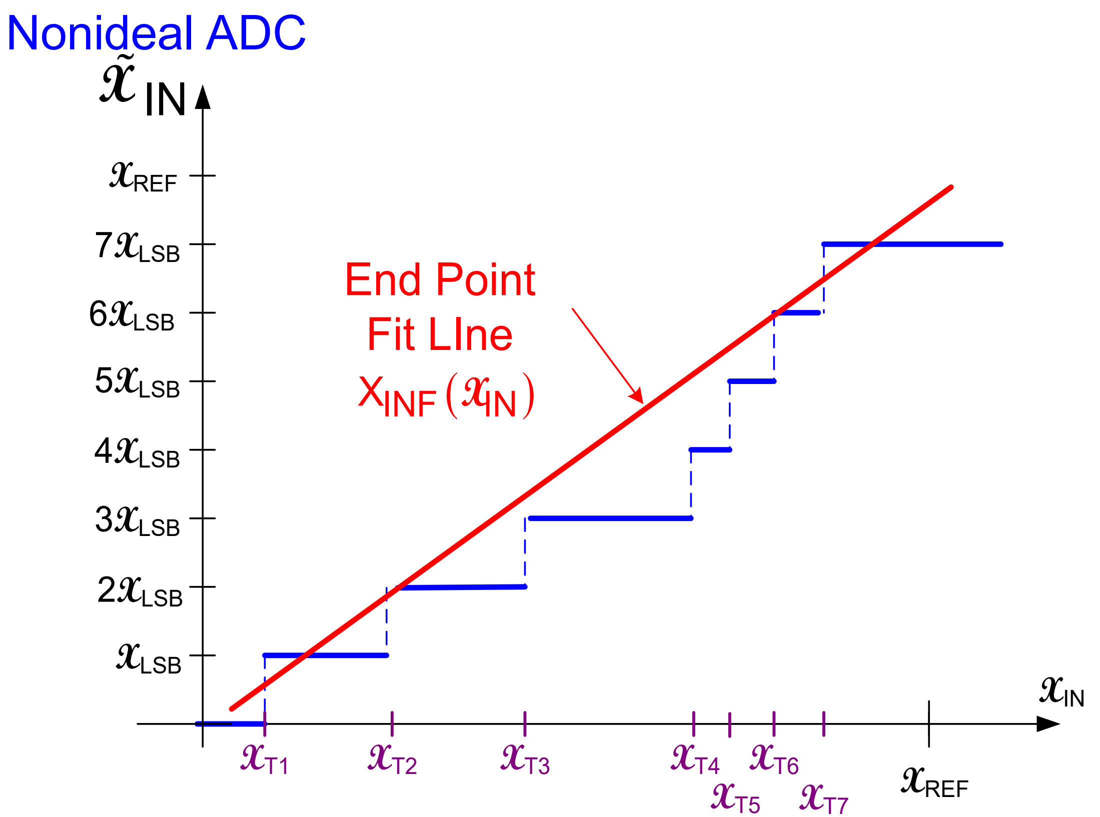



Transition points are not uniformly spaced !

More than one definition for INL exists !

Will give two definitions here

# Integral Nonlinearity (ADC)




Consider end-point fit line with interpreted output axis

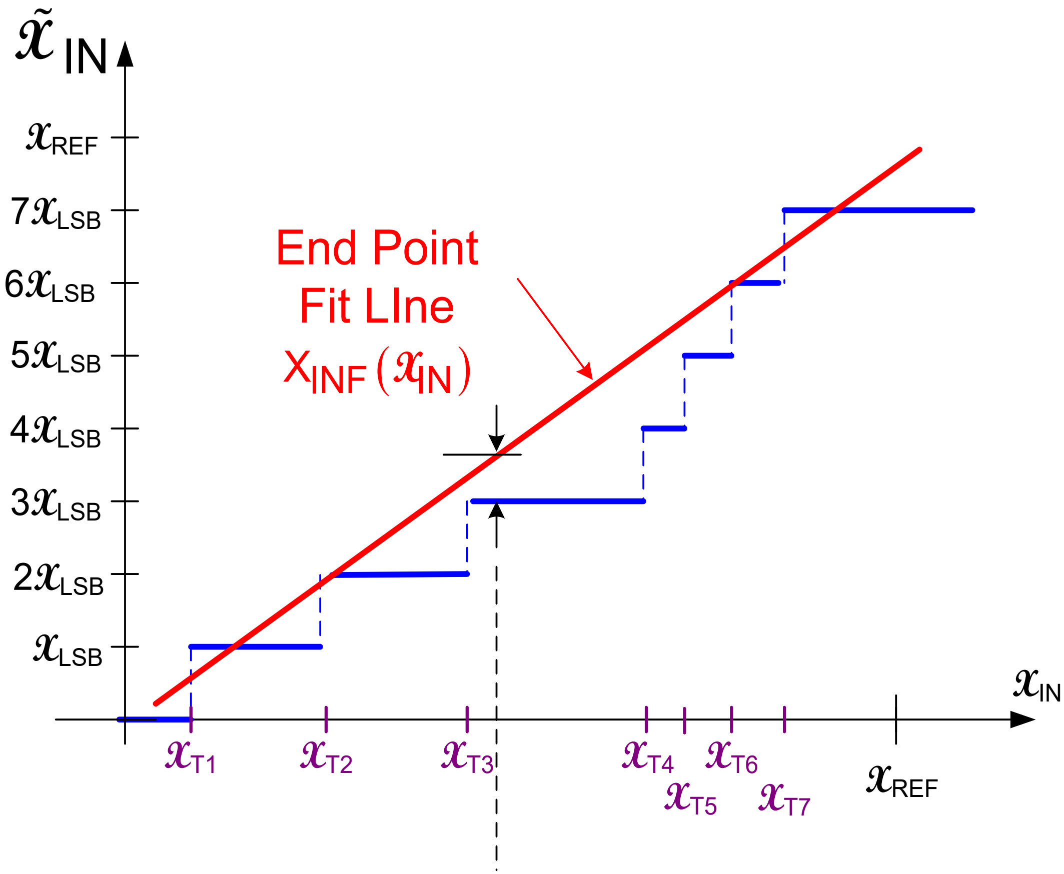
$$X_{INF}(x_{IN}) = m x_{IN} + \left( \frac{x_{LSB}}{2} - m x_{T1} \right)$$

$$m = \frac{(N-2)\, x_{LSB}}{x_{T7} - x_{T1}}$$

# Integral Nonlinearity (ADC)

Nonideal ADC

Continuous-input based INL definition



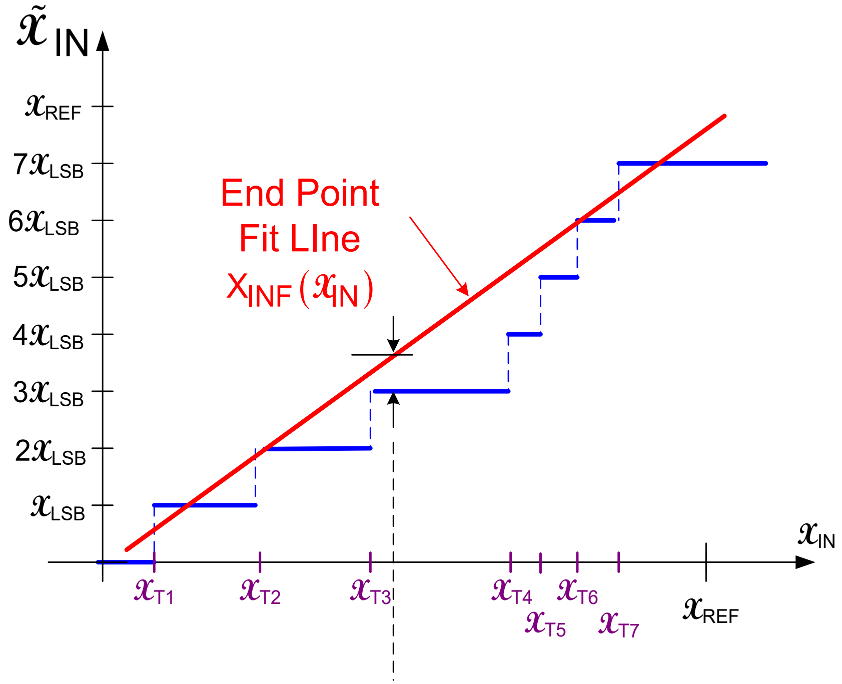

$$INL(x_{IN}) = \tilde{x}_{IN}(x_{IN}) - X_{INF}(x_{IN})$$

$$INL = \max_{0 \le x_{IN} \le x_{REF}} \left\{ \left| INL(x_{IN}) \right| \right\}$$

# Integral Nonlinearity (ADC)

Nonideal ADC

Continuous-input based INL definition



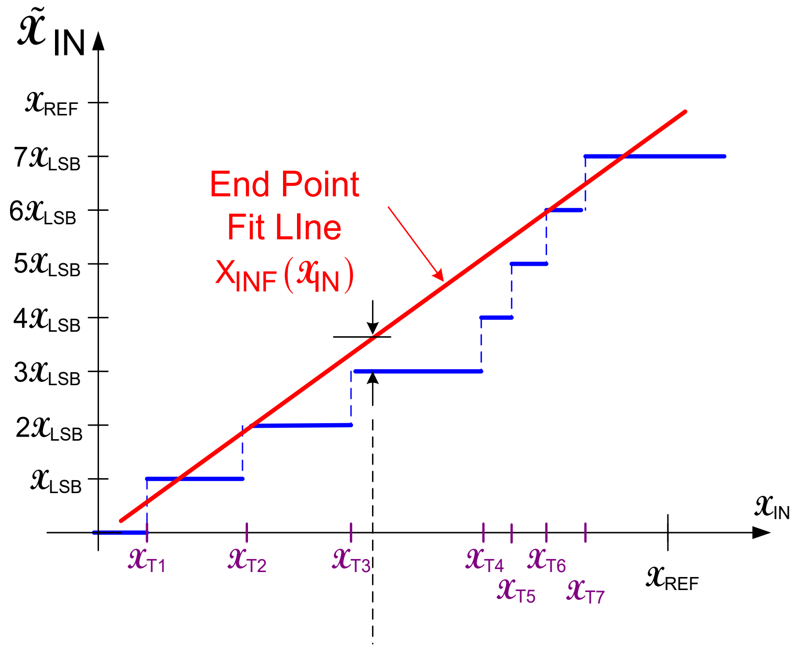

Often expressed in LSB

$$INL(x_{IN}) = \frac{\tilde{x}_{IN}(x_{IN}) - X_{INF}(x_{IN})}{x_{LSB}}$$

$$INL = \max_{0 \le x_{IN} \le x_{REF}} \left\{ \left| INL(x_{IN}) \right| \right\}$$

# Integral Nonlinearity (ADC)

Nonideal ADC



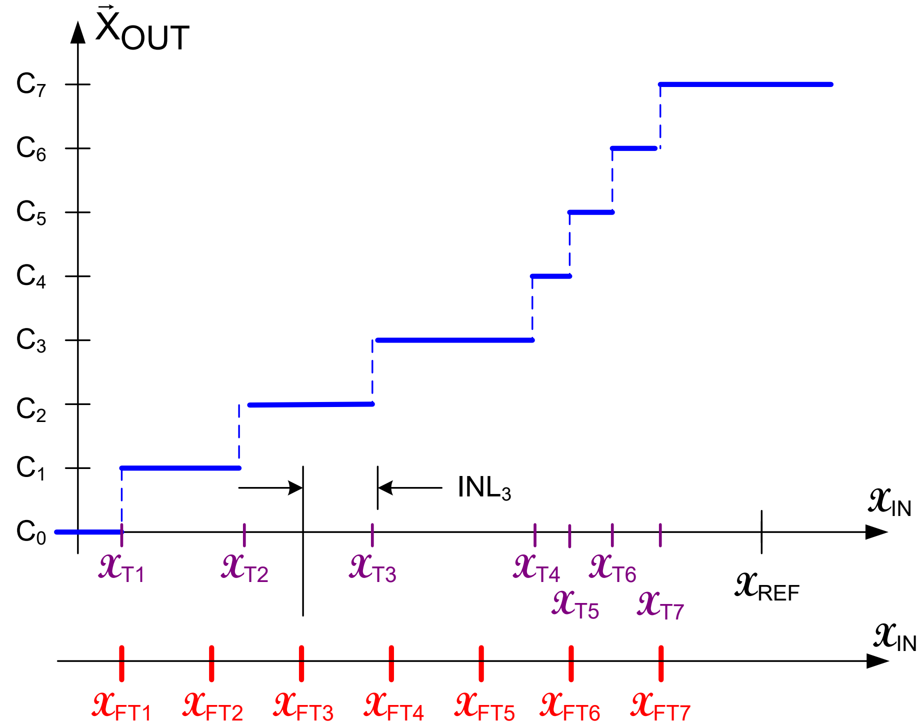

With this definition of INL, the INL of an ideal ADC is $x_{LSB}/2$ (for $x_{T1}=x_{LSB}$)

This is effective at characterizing the overall nonlinearity of the ADC but does not vanish when the ADC is ideal and the effects of the breakpoints are not explicit

# Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition (most popular)




Place N-3 uniformly spaced points between $X_{T1}$ and $X_{T(N-1)}$ designated $x_{FTk}$

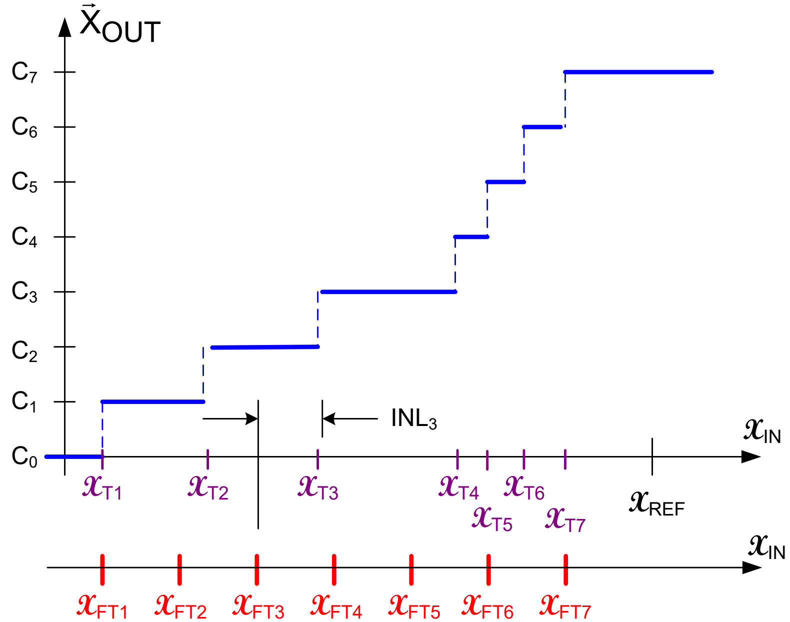
$$INL_k = x_{Tk} - x_{FTk} \qquad 1 \le k \le N\text{-}2$$

$$INL = \max_{2 \le k \le N-2} \{ |INL_k| \}$$

# Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition (assuming all break points present)




Often expressed in LSB

$$INL_k = \frac{x_{Tk} - x_{FTl}}{x_{LSB}} \qquad 1 \le k \le N-2$$

$$INL = \max_{2 \le k \le N-2} \{|INL_k|\}$$

For an ideal ADC, INL is ideally 0

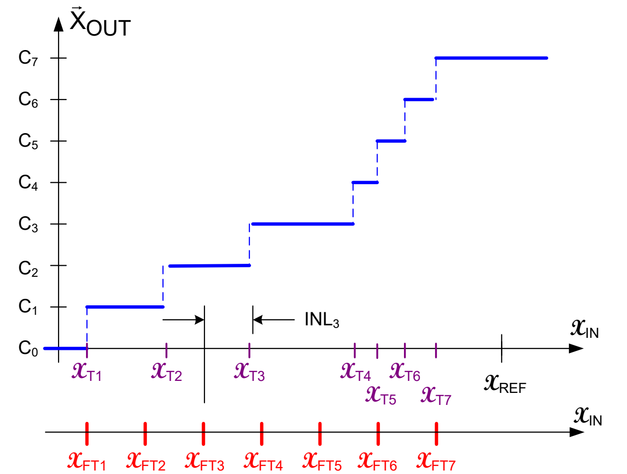
# Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

$$INL_k = \frac{x_{Tk} - x_{FTl}}{x_{LSB}} \qquad 1 \le k \le N-2$$

$$INL = \max_{2 \le k \le N-2} \{ |INL_k| \}$$




- INL is often the most important parameter of an ADC
- $INL_1$ and $INL_{N-1}$ are 0 (by definition)
- There are N-3 elements in the set of $INL_k$ that are of concern
- INL is a random variable at the design stage
- $INL_k$ is a random variable for $0<k<N-1$
- $INL_k$ and $INL_{k+j}$ are correlated for all k,j (not incl 0, N-1) for most architectures
- Fit Line (for cont INL) and uniformly spaced break pts (breakpoint INL) are random variables
- INL is the N-3 order statistic of a set of N-3 correlated random variables (breakpoint INL)

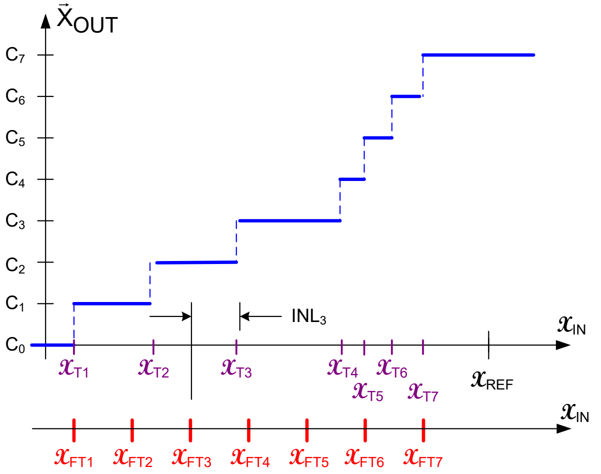
# Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

$$INL_k = \frac{x_{Tk} - x_{FTl}}{x_{LSB}} \qquad 1 \le k \le N\text{-}2$$

$$INL = \max_{2 \le k \le N\text{-}2} \{|INL_k|\}$$




- At design stage, INL characterized by standard deviation of the random variable
- Closed-form expressions for INL almost never exist because PDF of order statistics of correlated random variables is extremely complicated
- Simulation of INL very time consuming if n is very large (large sample size required to establish reasonable level of confidence)
  - -Model parameters become random variables
  - -Process parameters affect multiple model parameters causing model parameter correlation
  - -Simulation times can become very large

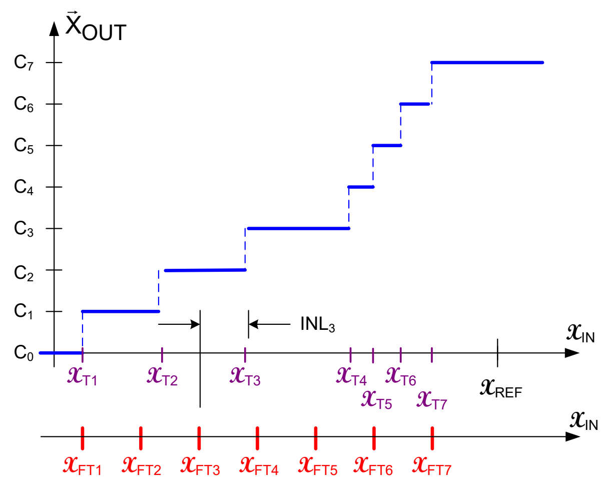
# Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

$$INL_k = \frac{x_{Tk} - x_{FTI}}{x_{LSB}} \qquad 1 \le k \le N-2$$

$$INL = \max_{2 \le k \le N-2} \{ |INL_k| \}$$




- INL can be readily measured in laboratory but often dominates test costs because of number of measurements needed when n is large
- INL is a random variable and is a major contributor to yield loss in many designs
- Expected value of $INL_k$ at k=(N-1)/2 is largest for many architectures
- This definition does not account for missing transitions
- Major effort in ADC design is in obtaining an acceptable yield

# INL-based ENOB

Consider initially the continuous INL definition for an ADC where the INL of an ideal ADC is $X_{LSB}/2$

Assume $$\text{INL}= \upsilon X_{LSBR}$$

where $X_{LSBR}$ is the LSB based upon the defined resolution, $n_R$

$$\text{INL}=\frac{\upsilon X_{REF}}{2^{n_R}} = \frac{X_{REF}}{2^{n_{eq}+1}}$$

Thus

$$\frac{\upsilon}{2^{n_R}} = \frac{1}{2^{n_{eq}+1}}$$

But $\text{ENOB}_{INL}=n_{eq}$

Hence

$$\text{ENOB} = n_R - 1 - \log_2(\upsilon)$$

# INL-based ENOB

$$ENOB = n_R - 1 - \log_2(\upsilon)$$

Consider an ADC with specified resolution of n (dropped the subscript R) and INL of v LSB

| $\upsilon$ | ENOB |
|---|---|
| ½ | n |
| 1 | n-1 |
| 2 | n-2 |
| 4 | n-3 |
| 8 | n-4 |
| 16 | n-5 |

# Performance Characterization of Data Converters

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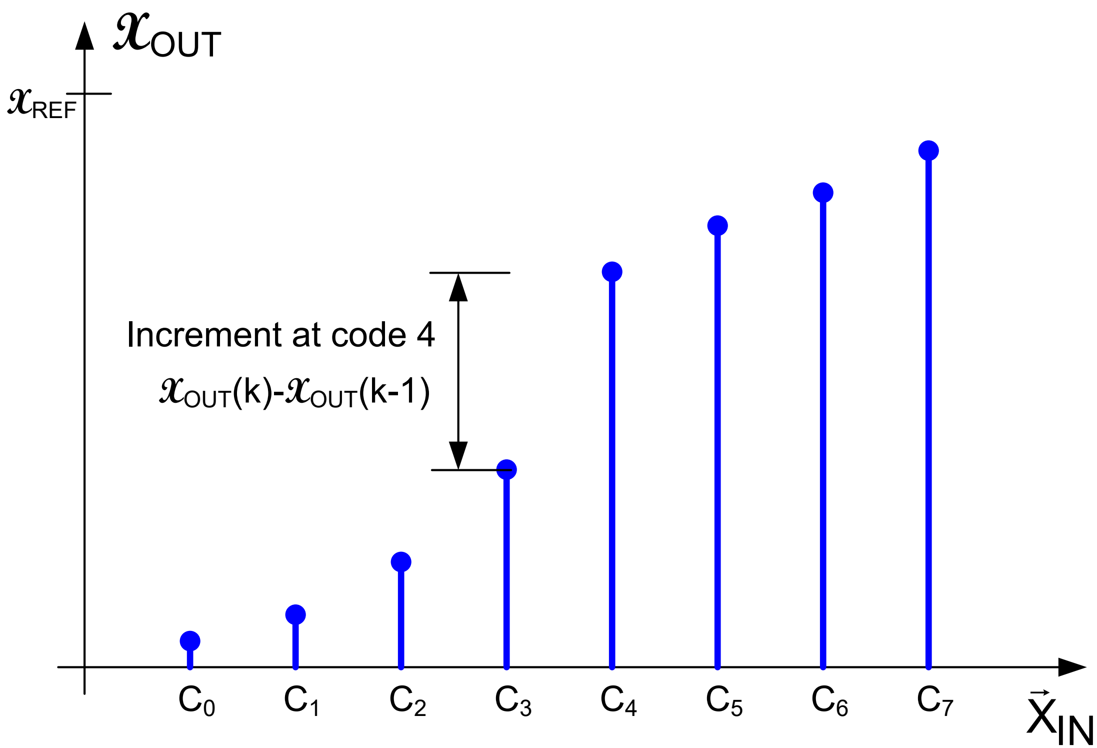
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

# Differential Nonlinearity (DAC)

Nonideal DAC

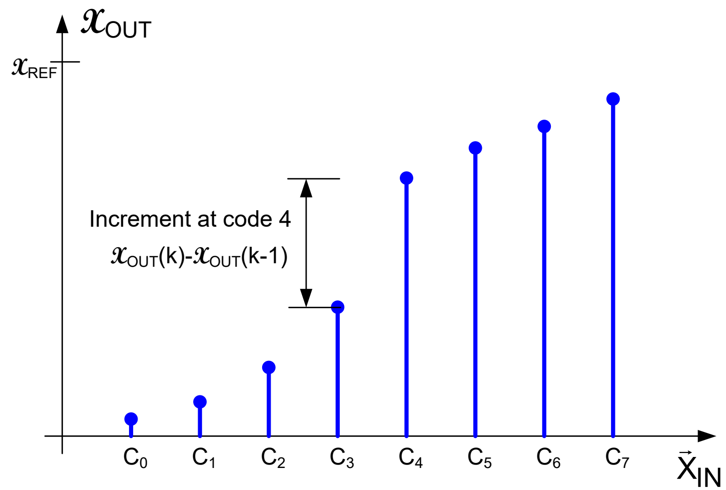



DNL(k) is the actual increment from code (k-1) to code k minus the ideal increment normalized to $X_{LSB}$

$$DNL(k)=\frac{X_{OUT}(k)-X_{OUT}(k-1)-X_{LSB}}{X_{LSB}}$$

# Differential Nonlinearity (DAC)

Nonideal DAC




Increment at code k is a signed quantity and will be negative if $X_{OUT}(k)<X_{OUT}(k-1)$

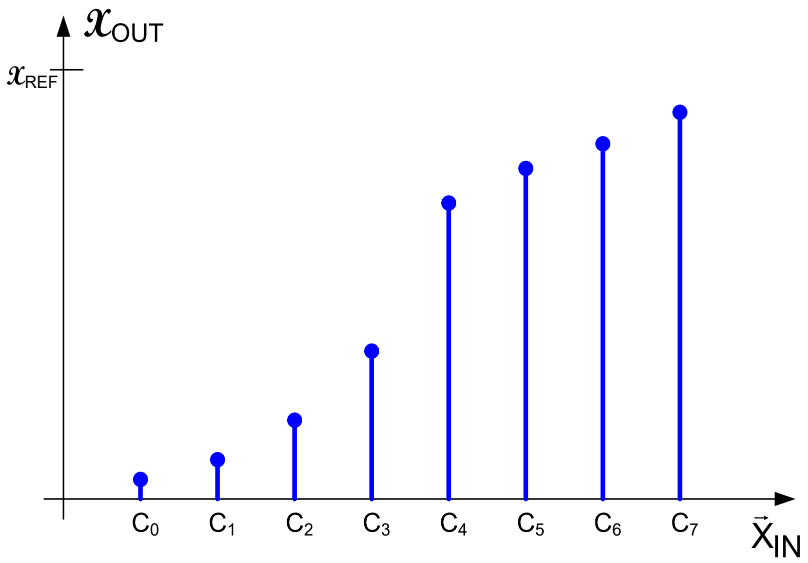
$$DNL(k)=\frac{X_{OUT}(k)-X_{OUT}(k-1)-X_{LSB}}{X_{LSB}}$$

$$DNL=\max_{1\leq k\leq N-1}\left\{\left|DNL(k)\right|\right\}$$

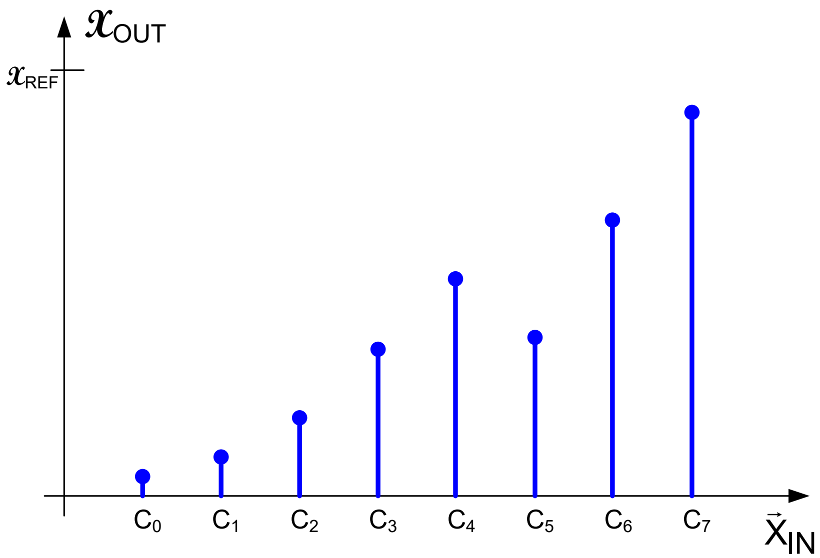
DNL=0 for an ideal DAC

# Monotonicity (DAC)

Nonideal DAC




Monotone DAC




Non-monotone DAC

Definition:

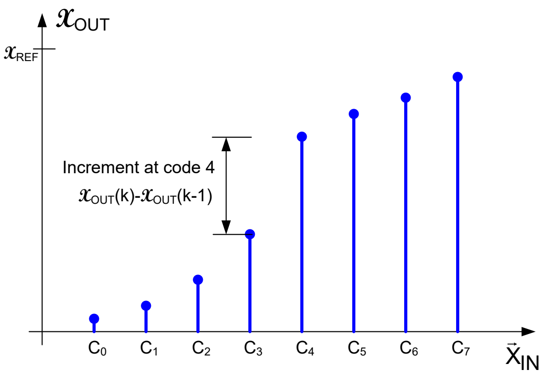
A DAC is monotone if $x_{OUT}(k) > x_{OUT}(k-1)$ for all k

Theorem:

A DAC is monotone if DNL(k)> -1 for all k

# Differential Nonlinearity (DAC)

Nonideal DAC




Theorem: The $INL_k$ of a DAC can be obtained from the DNL by the expression

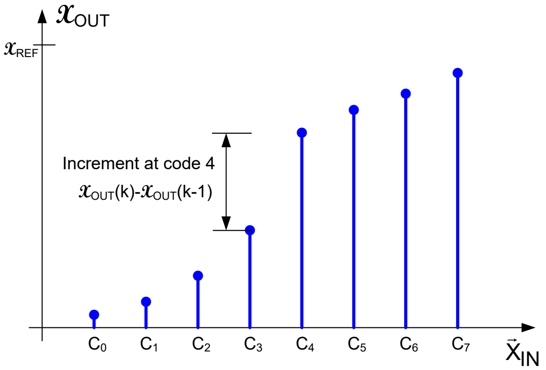
$$INL_k = \sum_{i=1}^{k} DNL(i)$$

Caution: Be careful about using this theorem to measure the INL since errors in DNL measurement (or simulation) can accumulate

Corollary: $DNL(k)=INL_k-INL_{k-1}$

# Differential Nonlinearity (DAC)

Nonideal DAC




Theorem: If the INL of a DAC satisfies the relationship

$$INL < \frac{1}{2} X_{LSB}$$

then the DAC is monotone

Note: This is a necessary but not sufficient condition for monotonicity

Stay Safe and Stay Healthy !

End of Lecture 26